

# SC 632 Variational Analysis and Polyhedral Geometry

## New course

This is a new and unique course concerning two topics – variational inequalities and the geometry of polyhedra. Common to both is the beautiful and intricate geometry of convex sets and the many ingenious concepts and techniques developed to understand it.

Variational inequalities are a class of versatile and widely applicable mathematical problems that unify diverse topics such as convex optimization, saddle point problems, economic equilibrium problems, continuous kernel Nash games, elastoplastic structural analysis, nonlinear obstacle problems and pricing of options in finance. The first part of this course aims to give a quick and thorough introduction to variational inequalities. The results we develop will take in their sweep many well known results from optimization and games. This course will begin from the basics of convex sets following which it will introduce powerful tools of nonlinear analysis and then proceed to probe the variational inequality problem. Specific topics are as follows.

- Introduction to Variational Inequalities – source problems, special cases and generalizations
- Solution analysis – existence, uniqueness, compactness of solution sets, etc
- Monotone and Cartesian variational inequalities
- Algorithms for computing solutions to variational inequalities.

Polyhedra are arguably the most well-studied convex sets, which also have interesting combinatorial properties. In this part we will explore the structure of polyhedra in detail, and explore some extensions to linear semi-infinite systems, i.e., the solutions to an arbitrary set of linear inequalities in finite-dimensional space. Specific topics include:

- Geometry of polyhedra
- The geometry of Semi-Infinite Programming (SIP)
- Semi-infinite duality
- Applications to Locally polyhedral sets

## References

- F. Facchinei and J-S Pang, Finite Dimensional Variational Inequalities - Vol I, 2003, Springer.
- M. A. Goberna and M. A. López, Linear Semi-Infinite Optimization, 1998, John Wiley.
- G. M. Ziegler, Lectures on Polytopes, 1995, Springer.

## Instructors

- Ankur A. Kulkarni, Systems and Control Engineering (variational inequalities)  
**Email:** [ankur@sc.iitb.ac.in](mailto:ankur@sc.iitb.ac.in) **Website:** <http://www.sc.iitb.ac.in/~ankur/>
- Vishnu Narayanan, Industrial Engineering and Operations Research (polyhedral geometry)  
**Email:** [vishnu@iitb.ac.in](mailto:vishnu@iitb.ac.in) **Website:** <http://www.ieor.iitb.ac.in/~vishnu/>

## Logistics

- **Credits:** 6
- **Slot:** 6, **Room:** LCT21. This information *may* change. Check <http://asc.iitb.ac.in> for the latest.
- **Prerequisites:** Knowledge of real analysis. Knowledge of optimization is desirable.